Calculating the HALO Ratio

The HALO Ratio is the number used to determine what size and weight of load a particular size of HALO is capable of turning.

The ratio is defined as follows:

HALO RATIO = MR^2 of the HALO ÷ MR^2 of the LOAD

Where M = Mass and R = Radius of Gyration.

Radius of Gyration, $Rg = \sqrt{\frac{I}{A}}$ where I = Area Moment of Inertia and A = Cross Sectional Area

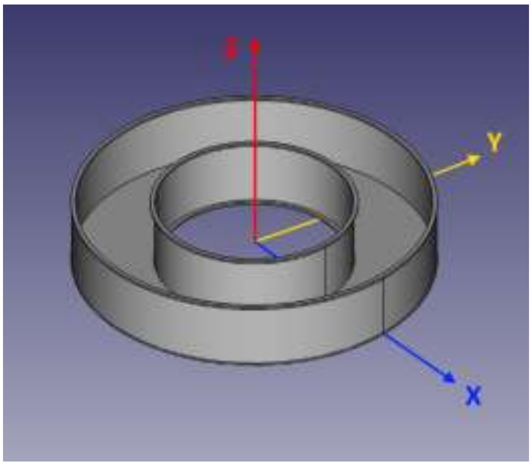
If the ratio, expressed as a percentage, is 3% or greater, it will turn the load.

NOTE: As the ratio goes above 3% the HALO becomes more powerful, and at around 6% will likely turn the load faster than required if full power is applied.

Radius of Gyration of the HALO

First we need to calculate the Area Moment of Inertia (*I*) of the HALO. This is with respect to rotation around the Z Axis as shown below.

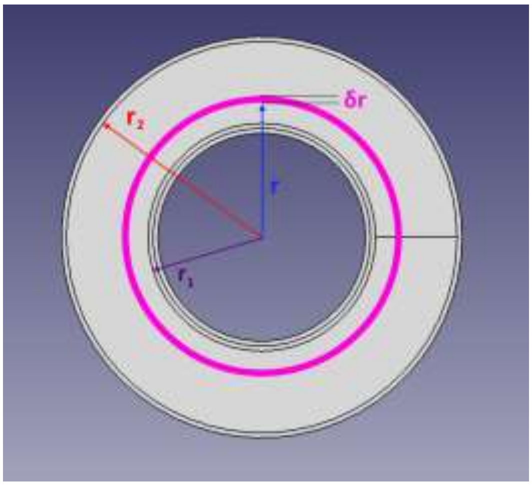
Since the HALO tank has a hollow centre, Area Moment of Inertia (*I*) is calculated as follows:



$$I = \int_{r_1}^{r_2} 2\pi r \delta r r^2 \, dA$$

$$I = 2\pi \int_{r_1}^{r_2} r^3 \delta r \; dA$$

$$I = \pi \frac{r_2^4 - r_1^4}{2}$$



Where:

An elemental circular area on the surface of the tank has a difference in radius of δr

r = radius from the centre to the elemental area

r_1 = radius to the inside wall of the tank

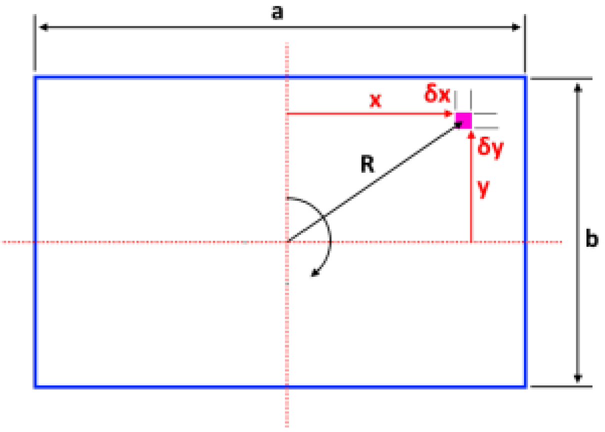
r_2 = radius to the outside wall of the tank

The Radius of Gyration is then:

$$Rg = \sqrt{\frac{\pi(r_2^4 - r_1^4)}{2(\pi r_2^2 - \pi r_1^2)}}$$

Radius of Gyration of the Load

If we consider a rectangular load, the Area Moment of Inertia (*I*) with respect to rotation around the central axis looking down on the load as shown in the diagram below is calculated as follows:



First calculate *I* for one quarter area (I_q), with length $= \frac{a}{2}$ and width $= \frac{b}{2}$

Taking once again an elemental area at radius R from the centre, length $= \delta x$ and width $= \delta y$ then:

$$I_q = \int_0^{b/2} \int_0^{a/2} (x^2 + y^2)\, dx\, dy$$

The first integral then becomes:

$$\left[\frac{x^3}{3} + y^2 x\right]_0^{\frac{a}{2}}$$

$$= \frac{(a/2)^3}{3} + y^2(a/2)$$

The second integral is then:

$$\int_0^{b/2} \frac{a^3}{24} + \frac{y^2 a}{2} dy$$

$$I_q = \left[\frac{a^3 y}{24} + \frac{y^3 a}{6}\right]_0^{\frac{b}{2}}$$

$$I_q = \frac{a^3 b}{48} + \frac{b^3 a}{48}$$

Now multiply by four to get *I* for the whole area:

$$I = \frac{a^3 b}{12} + \frac{b^3 a}{12}$$

$$Rg = \sqrt{\frac{a^3 b + b^3 a}{12ab}} = \sqrt{\frac{a^2 + b^2}{12}}$$

Radius of Gyration of a Circular Load

Radius of Gyration of a solid circular load can be calculated with the same formula as the HALO. If it is a solid cylinder, then $r_1 = 0$, which simplifies the formula to:

$$Rg = \sqrt{\frac{r^2}{2}}$$

HALO Ratio

The HALO Ratio can now be calculated once the mass of the liquid in the HALO is known and the mass of the load.

Note that the mass of the liquid in the HALO is dependent on any additives that may have been mixed into the water, so the resultant density needs to be taken into account, so the final formula is:

HALO RATIO (as a percentage) = (VDR^2 of the HALO ÷ MR^2 of the LOAD) × 100

Where:

V = Volume of Liquid
D = Density of Liquid
M = Mass of Load
R = Radius of Gyration

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